A Matching-Unscented Kalman Filtering for Gravity Aided Navigation

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ABSTRACT

A matching-unscented Kalman filtering for gravity aided navigation is presented in this paper. With this method submerged position fixes for autonomous underwater vehicle can be obtained from comparing gravity fields’ measurements with gravity maps, meanwhile the drawback of traditional matching or filtering algorithms can be avoided. A synthetic gravity map was taken for the simulation, and the results showed that navigation errors can be reduced more efficiently and reliably by the presented method.

Keywords: gravity navigation, underwater navigation, gravity matching, unscented Kalman filter

1. INTRODUCTION

Inertial navigation systems (INSs) are widely used on autonomous underwater vehicles (AUVs) to establish underwater navigation system. However, the navigation error of INS is increased with time during extended AUV missions. One method for correcting such errors without compromising the AUV mission is by use of underwater gravity aided navigation. Considering the covertness of an autonomous underwater vehicle (AUV), gravimetry is not easily detected and interfered with, unlike radar, laser and sonar. It does not need an AUV to float near the surface and is non-emanating and so completely covert[1-4].

Researching of stable precise location algorithm is one of the most important key technologies of gravity aided navigation system. Some matching and filtering algorithms for gravity aided navigation were developed in the past. With matching algorithms like TERCOM or ICCP, large navigation errors could be corrected, but a sequence of gravity measurements need to be collected in advance. So the matching process can not be simultaneous[5-6]. The extended Kalman filtering (EKF) can provide continuous updating of position and velocity information of a vehicle. But its sensitivity to the initial position error and the linearization error usually results in divergence. So the reliability and stability can not be ensured in filtering process[7-9].

For superior alternative to EKF, unscented Kalman filtering (UKF) has been used greatly in road vehicle navigation, tracking and neural network[10-15]. In this paper, a matching-unscented Kalman filtering (MUKF) is proposed for gravity aided navigation. With this method drawback of traditional matching or filtering algorithms can be avoided, submerged position fixes for autonomous underwater vehicle can be obtained from comparing gravity fields’ measurements with gravity maps. Simulation results show that navigation errors can be reduced more efficiently and reliably by the presented method.

The structure of this paper is as follows. In section 2 the MUKF based navigation system is introduced and the MUKF based navigation algorithm is explored in section 3. Simulation results are discussed in section 4 and the conclusion is provided in section 5.

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2. MUKF BASED GRAVITY AIDED NAVIGATION SYSTEM

A matching-unscented Kalman filtering (MUKF) for gravity aided navigation is presented in this paper. The framework of the whole system is illustrated in Fig.1.

From this figure it can be seen that, digital gravity map was stored and the track was planned before the AUV mission. As the AUV moves, system gathers a set of gravity measurement and compared this set to various data sets from reference map. The optimal fit is the path we need. With this matching process, navigation errors can be reduced, and a preliminary location result could be gotten. This preliminary result obtained in matching process can be introduced as initial position into filtering process. The reliability and stability of filtering would be improved because of the preliminary result is close to the actual that the initial position error is relatively small. And in most cases, the final location result of filtering is more accurate than the preliminary result.

3. UNSCENTED KALMAN FILTERING

When the vehicle is in a state of motion, the values of gravity are measured at discrete points simultaneously using a gravimeter fixed on vehicles. When the preliminary location result is gotten through matching process, we wish to estimate the position of the vehicle in real-time through filtering process.

An approach to modelling the system is

\[
\begin{align*}
    x_{t+1} &= \Phi(t)x_t + w_t \\
    y_{t+1} &= H(x_{t+1}) + n_{t+1}
\end{align*}
\]

(1)

The first equation describes the propagation of the system’s state in time.

Where \( x_t = [\delta r_x(t), \delta r_y(t), \delta r_z(t), \delta v_x(t), \delta v_y(t), \delta v_z(t)]^T \) is the state of the system at timestep t, the symbol \( \delta \) is defined as the error between the actual state and the referenced state which presented by INS:

\[
\delta x_u = (x_u)_{actual} - (x_u)_{INS}
\]

\( x_u = r_x, r_y, r_z, v_x, v_y \) or \( v_z \)
\((r_x, r_y, r_z)\) is the coordinates of the vehicle's position in the Cartesian reference frame, \(v_x\), \(v_y\) and \(v_z\) is velocity components of the vehicle at \(x\), \(y\) and \(z\) orientations respectively; \(\Phi(t)\) is the process noise of the dynamic system given by \(\nu(t)\).

The second equation in formula (1) is the measurement equation. Where \(y = \delta g = g_{\text{actual}} - g_{\text{INS}}\), the symbol \(\delta\) is also defined as the error between the actual gravity measurement values and the referenced gravity values, which are given according to the INS state and the underwater gravity map. \(H(x_{\nu})\) with a measurement error of \(n(t)\).

It is assumed that the additive noise vectors, \(\nu(t)\) and \(n(t)\) are Gaussian, uncorrelated white sequences.

With UKF, the system’s state can be accurately estimated, finally the navigation error of INS can be corrected. The UKF equations for UGAN are given as follows:

(a) Initialize with:

\[
\begin{bmatrix}
\hat{x}_0 \\
\hat{p}_0 \\
\hat{e}_0
\end{bmatrix} = \begin{bmatrix}
E[x_0] \\
E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T] \\
E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T]
\end{bmatrix}
\]

(b) Calculate sigma points with corresponding weights:

\[
\begin{align*}
(x^*)_h &= \hat{x}_0 + W_{-}^{\alpha} (\hat{x}_0 - \hat{p}_0) \\
(x^*)_i &= \hat{x}_0 + \sqrt{L + \lambda} \hat{p}_0, & i = 1, \ldots, L \\
(x^*)_c &= \hat{x}_0 - \sqrt{L + \lambda} \hat{p}_0, & i = L + 1, \ldots, 2L \\
W_{-}^{\alpha} &= \frac{1}{L + \lambda} \\
W_{+}^{\alpha} &= \frac{1}{2(L + \lambda)} \\
W_{-}^{\beta} &= \frac{\lambda}{L + \lambda} \\
W_{+}^{\beta} &= \frac{1}{2L + \lambda}
\end{align*}
\]

(c) Time update:

\[
\begin{align*}
\hat{x}_{t+1} &= \Phi(t) \hat{x}_t + \mu(t) \\
\hat{p}_{t+1} &= \Phi(t) \hat{p}_t + \nu(t) \\
\hat{e}_{t+1} &= \Phi(t) \hat{e}_t
\end{align*}
\]

(d) Measurement update:
\[
\begin{align*}
\mathbf{P}_{t+1|t} &= \sum_{i=1}^{2N} W^i \mathbf{(x}_{t+1|t} - \mathbf{x}_{t+1|t}) (\mathbf{x}_{t+1|t} - \mathbf{x}_{t+1|t})^T, \\
\mathbf{S}_{t+1} &= \mathbf{P}_{t+1|t} - \sum_{i=1}^{2N} W^i \mathbf{(x}_{t+1|t} - \mathbf{x}_{t+1|t}) (\mathbf{x}_{t+1|t} - \mathbf{x}_{t+1|t})^T, \\
\mathbf{x}_{t+1} &= \mathbf{x}_{t+1} + \mathbf{S}_{t+1} (\mathbf{y}_{t+1} - \mathbf{y}_{t+1}), \\
\mathbf{P}_{t+1} &= \mathbf{P}_{t+1} - \mathbf{S}_{t+1} \mathbf{P}_{t+1}.
\end{align*}
\] (5)

Where \( \lambda = \alpha^2(L + \kappa) - L \) is a scaling parameter. \( \alpha \) determines the spread of the sigma points around \( \mathbf{x} \) and is usually set to a small positive value (e.g., 10^{-3}). \( \kappa \) is a secondary scaling parameter which is usually set to 0, and \( \beta \) is used to incorporate prior knowledge of the distribution of \( \mathbf{x} \). \((\sqrt{(L + \lambda)} \mathbf{P}_{t}^\lambda \)) is the \( i \) th row of the matrix square root.

In EKF, the observation model \( \mathbf{H} \) in equation (1) is linearized so the system is approximated by a linear system. However, large errors can be introduced in this first-order approximation.

In the previous UKF, \( \mathbf{H} \) could be formulated through a kinematic model. But in particular, the values of \( y_{t+1|t} \) here in equation (4) \( y_{t+1|t} = H(\mathbf{x}_{t+1|t}, \mathbf{x}_{t+1|t}) \) could only be gotten from the gravitational maps with corresponding coordinates of the vehicle’s position which can be calculated with \( \mathbf{x}_{t+1|t} \) and the state of INS, although no expression of \( \mathbf{H} \) can be constructed here.

## 4. SIMULATION RESULTS AND DISCUSSION

This section applies and compares the performance of the MUKF against a TERCOM and an EKF algorithm for the gravity aided navigation system. We wish to estimate the location error, which can be used to modify the INS.

A synthetic gravity map with 1.5' * 1.5' sizing grid was taken for the simulation. Firstly, independent TERCOM or Extended Kalman Filtering (EKF) was used for gravity aided navigation, so a location result could be obtained by each one algorithm respectively. These two location results can be prepared for comparison.

Then the presented MUKF method was introduced into navigation system. Inside TERCOM method was chosen for matching process, in which Minimum Square Distance (MSD) algorithm was used, and UKF was chosen for filtering process.

<table>
<thead>
<tr>
<th>Table 1 Statistical results of three methods</th>
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<tbody>
<tr>
<td>Method</td>
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<tr>
<td>--------</td>
</tr>
<tr>
<td>TERCOM</td>
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<tr>
<td>EKF</td>
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<tr>
<td>MUKF</td>
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</tbody>
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After multiple simulation tests, the average location errors and valid-matching/ convergence filtering probabilities which committed by three methods across a Monte Carlo simulation consisting of 100 runs was calculated, the results are showed in Table 1. From the table it can be found that, independent TERCOM matching method got a 2.15 n mile average location error, and a 90.0% valid-matching probability. Besides, independent EKF had a 4.57 n mile average location error, and a 75.8% convergence probability. By contrast, 1.22 n mile average location error and 90.8% convergence probability was reached by MUKF method.

In comparison, the difference of accuracy and stability between MUKF and traditional algorithms is very apparent. Therefore, we conclude that in this example of underwater gravity-aided navigation the MUKF has substantial advantages over the TERCOM and EKF in performance.
5. CONCLUSION

A matching-unscented Kalman filtering (MUKF) for gravity aided navigation is explored and discussed in this paper. In comparison with some traditional matching or filtering algorithms like TERCOM and Kalman Filtering, MUKF is more simultaneous, accurate and stable. The synthetic gravity map was chosen for simulation, and results show that navigation errors can be reduced more efficiently and reliably by the presented method.

REFERENCES